TESTING FOR NONLINEAR DYNAMICS IN THE STOCK EXCHANGE OF THAILAND (SET)

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Abstract

Understanding stock market price fluctuations plays an important role in economic policy and in corporate investment and financing strategies. In recent years, Khantavit and others have investigated the proposition that nonlinear processes studied in Chaos theory play an important role in these fluctuations. This study provides a detailed examination of this hypothesis using data from the Stock Exchange of Thailand (SET) from 1975 to 1999. The study finds that the distribution of the daily return on the SET index is non-normal and leptokurtic. The results of the study also suggest that non-linear processes play a significant role in stock market behavior

I. INTRODUCTION

Consider the recent behavior of the Thai stock market, officially called The Stock Exchange of Thailand or SET. In 1997 the SET shed 54% of its beginning value for the year to close at 372.69 on December 31. One month

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later, in January 1998, the market soared by about 35% from 366.18 to close at 495.23 on January 30, 1998. On the following trading day, February 2, 1998, the SET index jumped another12.02% to close at 554.75 only to reverse its direction two days later on February 4, 1998, when it closed down by about 9%. Figure 1 shows the daily movement of the SET index from April 30, 1975, the day the SET began operations, to December 30, 1999 (a period of approximately 25 years). What explains the apparent erratic movement of the SET index? Hsieh (1991) suggests there are a number of explanations and the popular one is that the stock market is governed by chaotic dynamics.

This research examines the changes in the Stock Exchange of Thailand (SET)'s general index from April 30, 1975 through December 31, 1999. It attempts to test the daily return series for a chaotic behavior that would explain the seemingly random fluctuation in the capital market. However, the study first conducts tests for normality and linearity. If non-linearity is detected, it seeks to describe the properties of the underlying structure using chaos theory applications.

The study expands Khanthavit (1995) in two important ways. First, it examines longer period and includes more than three times as much data points. The study examines the SET index behavior from April 30, 1975, the day The Stock Exchange of Thailand began its operation and started the calculation of the index, to December 30, 1999. The extant methods used for studies on nonlinear dynamics are data-intensive. For example, the method of correlation dimension proposed by Grassberger and Procaccia requires a substantial number of data points (Hsieh 1991). Similarly, the rescaled range (R/S) analysis is highly data-intensive (Peters 1994). However, examining a longer period to obtain more data points would lead to the problem of nonstationarity. According to Khanthavit (1995, pp. 45) "a longer sample period is more likely to capture many more patterns [in the time series] and this in turn leads to a larger [correlation dimension] D_c (m =10)." Second, the study applies additional methods, for example, the R/S analysis and Hurst Exponent popularized by Peters (1989, 1991, and 1994), to further the analysis and reconfirm the findings.

The paper is structured as follows: Section II is a brief review of related literature and section III presents the methodology and data; Section IV presents the descriptive statistics of the data and results of the application of nonlinear dynamics techniques. Section V concludes the findings of the study.

II. **Brief Review of Related Studies**

The classical view of the market attributes any unusual changes in

Figure 1: The time-series representing the historical daily movement of The Stock Exchange of Thailand index : April 30, 1975 – December 30, 1999.

capital asset prices to random shocks caused by exogenous factors. “Fundamental value, one supposes, should be more stable. And prices are not in fact entirely random: they seem anchored, albeit elastically, to some underlying notion of worth.”[1] Larrain (1991) concludes nonlinear dynamic techniques (chaos theory) show that financial asset price behavior is not random at all [but] only appear to be random. Chaos theory suggests that capital markets are long-memory processes and are capable of producing endogenously large movements in prices that seem to change randomly although they are actually deterministic.

[1] The Economist, “Schools Brief: Shared values”, December 4th 1999, PP. 85.

Pandey (1998b) described the SET as one of the world's worst performers in 1997 and one of the best in the first quarter of 1998. There is no doubt that the Thai stock market has experienced wide fluctuations and seen its worst times in its recent history. But is the observed volatility of the SET unusual? Perhaps so for the 12.02% jump of the index in a single day on February 2, 1998, given the plus or minus 10% daily volatility band that existed until late 1997. And sure the SET index touched its lowest level in 10 years on September 4, 1998. However, we cannot consider these observations as unusual if we extend the period of examination and look at longer histories. The index closed at 76.43 on March 17, 1976 – its lowest point on record; its highest value on record is 1753.73 on January 4, 1994. Moreover, Khanthavit (1995) reminds us that during the second boom period [late 1980s and early 1990s] "the Thai market became very active and was affected by internal and external shocks such as the liberalization of money and capital markets, Black Monday, the Gulf War, and Thailand's May [1991] coup d'etat."

The science of model building consists of a set of quantitative tools, which are used to construct and then test mathematical representations of the real world (Bindyck and Rubinfeld 1991). Financial economists, working with hard data, have been attempting to model the return-generating process in the capital market since early 1950s. The efficient market hypothesis (EMH) is the bedrock upon which standard statistical analysis of the market has been built (Peters 1991) and linear modeling has been the traditional approach to identify the determinants of stock returns or model the capital market behavior. The capital market theory and its associated theoretical models in finance such as the traditional capital asset pricing model and option pricing model are based on the independent and identically distributed (IID) normality and linearity assumptions.

The linear view of the capital market implies that a certain change in the explanatory factors would lead to a proportionate change in stock return. For instance, the EMH holds that prices in the capital market adjust instantaneously to reflect the arrival of any new information. A major implication of EMH is that there is no significant correlation between returns. New information comes to the market randomly and stock prices are assumed to change proportionately and in a random fashion in reaction to the new information.

The IID normality and linearity assumptions are simple and convenient for the development of coherent financial models. However, the conclusions derived from such models are useful in describing the market behavior and in making short-term forecasting to the extent that their underlying assumptions are consistent with the facts. In other words, a standard of appropriateness calls for tests that can indicate whether statistical methods and the data to which they are applied are appropriate to each other (Neuburger and Stokes 1991).

A time-series model accounts for patterns in the past movements of a variable and uses that information to predict its future movements (Pindyck and Rubinfeld 1991). A financial time series is said to be normal if its distribution is approximately similar to the bell-shaped theoretical distribution and linear if a model involving only first power on all the predictor variables can explain its underlying structure. Contrary to the established IID normality and linearity assumptions, research on stock prices finds the distribution is leptokurtotic (Brorsen and Yang 1994). Higher peaks relative to the normal distribution and fat tails characterize a Leptokurtotic distribution.

According to Pandey (1998a) “it is widely known that Thai retail investors don’t discount news until it really takes place.” Such investors may not be able or have the time to digest the news properly and assess the potential impact on stock values. The result is that many retail investors simply follow the crowd, panicking or overreacting to dramatic events. De Bondt and Thaler (1985) study the Center for Research in Security Prices (CRSP) monthly return data and find evidence supporting the hypothesis that people’s overreaction to unexpected and dramatic news events affects stock prices.

Damodaran (1993) estimates the price adjustment coefficients for firms listed on various stock markets in the U.S. and reports evidence of lagged adjustment to new information in shorter return intervals. Damodaran (1993) indicates prices adjust much more slowly and with more noise for smaller firms. Neuburger and Stokes (1991) apply Hinich bispectrum to test the monthly Ibbotson Associates common stock and corporate bond return series from January 1926 through June 1988 for both normality and linearity. They find evidence of non-normality and non-linearity in both the stock and corporate bond return series.

Scheinkman and LeBaron (1989) develop algorithms and related statistical tests useful in distinguishing simple deterministic systems from stochastic systems. They apply their procedure to weekly and daily stock returns to find presence of nonlinear dependence. Khanthavit (1995) examines 2001 observations on Thailand’s stock prices for the period January 2, 1986 to February 10, 1994 and finds evidence of non-linear dependence consistent with low-dimension chaos in prices and returns. Nevertheless, Khanthavit (1995) attributes the findings and the failure of forecasts of stock returns based on neural networks to noise in the construction of the returns.

The results of the above studies suggest that models based on linearity and normality assumptions are not valid. The frequency of large movements in the daily SET index is larger than would be expected from a linear system. Therefore, the application of linear models to the market is questionable, in view of recent research suggesting that capital markets, and the economy as a whole, may be governed in part by nonlinear dynamics (Peters 1991). The stylized facts about the empirical distribution of stock prices have encouraged the development of alternative hypotheses such as the Coherent Market Hypothesis (CMH) and Fractal Market Hypothesis (FMH)[2].

III. **The Methodology and Data**

The methodology follows the lead of Grassberger-Procaccia (1983), Brock, Dechert, and Scheinkman (1987), Brock (1988), Scheinkman and LeBaron (1989), Hsieh (1991), and Peters (1989, 1991, and 1994). The study applies such popular nonlinear dynamics techniques as the BDS test, phase-plot construction, correlation dimension, Lyapunov exponent estimation, and rescaled range (R/S) analysis.

This study tests for nonlinear dependence between the data points by computing the BDS statistic proposed by Brock, Dechert, and Scheinkman (1987). The BDS statistic is based on the concept of correlation integral. Under the BDS method, the null hypothesis that data are independently and identically distributed is tested, that is, the data points do not influence each other and they are equally likely to occur. A rejection of the null hypothesis is consistent with some type of dependence in the data (Hsieh 1991).

According to Hsieh (1991), a generic property of chaotic processes is that chaotic maps do not fill up enough space in high dimension. So one way of detecting chaotic behavior in our return series is to first construct a phase space of the hidden return-generating process. And then proceed to describe the characteristics of the underlying structure or “attractor” by computing its correlation dimension. The correlation dimension is a measure of how much space is filled up by a string of data (Hsieh 1991, pp. 1847). Peters (1991) uses the concept of fractal dimension to measure how an attractor fills its space.

[2] The CMH holds that the state of the market changes over time and is determined by a combination of fundamental or economic factors and group sentiment. Accordingly the market can be in one of four states: a) coherence, b) chaos, c) unstable transition, and d) random walk. For more discussion of the CMH, see Vaga (1991).
The FMH holds that a market is stable if the investors’ investment time horizons and information sets are different. With uniform investment horizons and information, a market becomes unstable since investors are trading on the same information. See Peters, Edgar E., 1994, Fractal Market Analysis, New York: Wiley.

Estimating correlation dimensions and Lyapunov exponents are now-standard metric procedures for empirical studies (Gilmore 1993).

Alligood et al. (1996, pp. 106) state that "a characteristic of chaotic orbits is sensitive dependence on initial condition – the eventual separation of the orbits of nearby initial conditions as the system moves forward in time." The largest Lyapunov exponent is commonly used to measure this phenomenon. Alligood et al. (1996) define Lyapunov number as the average per-step divergence rate of nearby points along the orbit, and the Lyapunov exponent to be the natural logarithm of the Lyapunov number.

The R/S analysis was first developed by H.E. Hurst in the 1950s to model the Nile River's overflows over a period of time but its application to capital markets was popularized by Edgar E. Peters (1989, 1991, and 1994). We apply the R/S technique to analyze a series of daily logarithmic returns generated as follows:

$$x_t = \log (SET_t/SET_{t-1})$$

Where, x_t = logarithmic return or yield at time t
SET_t = closing price of the SET index at time t

Nonstationarity and autocorrelation bias tests of nonlinear dynamic (Hsieh 1991, Peters 1994). So we use logarithm to make the data stationary and accurately measure the price changes. Stock returns are said to be nonstationary if there are trends in the mean or variance. Moreover, since our interest is in detecting the existence (or lack of it) of nonlinear dependence, we filter the data by regressing x_t as the dependent variable against x_{t-p}, the independent variable, using autoregressive process of order p or AR (p) to obtain the equation,

$$x_t = \beta_1 x_{t-1} + \beta_2 x_{t-2} + \ldots + \beta_p x_{t-p} + e_t$$

Where, α = a constant and $|\alpha| \leq 1$
β = a constant and $|\beta| \leq 1$
e = a white noise

We then filter out the linear dependence and take the AR(p) residuals, y_t, such that

$$y_t = x_t - (\beta_1 x_{t-1} + \beta_2 x_{t-2} + \ldots + \beta_p x_{t-p})$$

This process eliminates or at least reduces serial correlation or short-memory effects between the data points. The order of the autoregressive process is chosen following the Akaike or Schwarz information criterion (AIC or SIC). Khanthavit (1995) chooses AR (8) for the return series based on the minimum Schwarz information criterion (SIC). Peters (1994, pp. 108-109) reports that "Brock, Dechert, and Scheinkman (1987) felt that it [an AR (1) residuals method] eliminated enough dependence to reduce the effect [of linear dependence] to insignificant levels, even if the AR process is level 2 or 3."

Peters (1992, pp. 81) states that "R/S analysis measures the cumulative deviation from the mean for various periods of time and examines how the range of this deviation scales over time." Mathematically,

$$R/S = (aN)^H$$

$$\text{Log } (R/S) = \log a + H \log N$$

Where,

R/S = rescaled range,

a = a constant,

N = number of observations,

and H = Hurst exponent

IV. **Results of the Data Analysis**

A) **Testing the Data for Stationarity**

The first step of the analysis involves preparing the data for use in testing for low dimensional chaotic behavior. We compute the daily logarithmic return on the SET to create the original time series. Changes in the logarithms of a variable produce better measurements of percent changes. Figure 2 is the plot of the computed original time series. An examination of the plot does not show any increasing or decreasing pattern in the time series, which implies the data are stationary. Nevertheless, a standard statistical analysis is necessary to ensure stationarity. Stationarity of the properties of the return-generating process would allow us to use past observations and develop a mathematical model of the process.

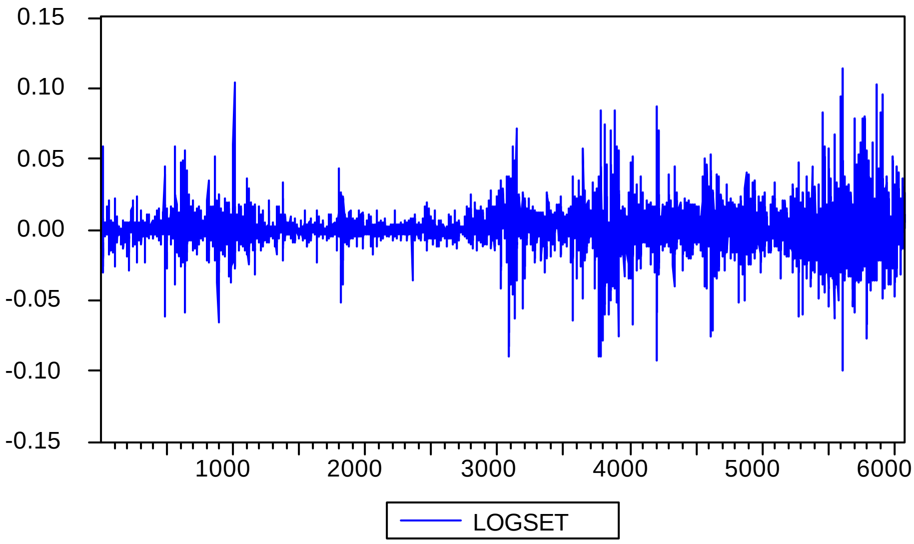

Figure 2: Line graph of unfiltered, logarithmic return of the daily SET index: April 30, 1975 December 30, 1999

Statistic	Computed value	Critical value
ADF test statistic (Lag = 10, and with trend and intercept)	-21.16	-3.41
PP test statistic (Newey-West suggested lag = 9, and with trend and intercept)	-64.00	-3.41

Table 1: Results of tests for stationarity.

Table 1 gives the result of Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests on the original series as well as the MacKinnon critical values for rejection of the hypothesis of the existence of a unit root at the 5% level of significance.

Since the ADF and PP test statistics are larger in absolute values than the critical values, we reject the hypothesis of nonstationarity. This confirms the finding of a visual inspection of the plot in figure 2. We therefore conclude that the original time series, i.e., the daily logarithmic return on the SET index, is stationary.

B) **Filtering the Series**

The purpose of the study is to test the daily return on SET index for nonlinear dynamics, particularly low-dimensional chaos. We therefore attempt to remove any linear dependence in the time series. Baumol and Benhabib (1989) recommend filtering by fitting an ordinary least squares (OLS) or autoregressive models to the time series.

Based on the selections of both the Akaike and Schwarz information criteria, an AR(6) model was fit to the data and then the residuals taken to remove or at least minimize the effect of any linear dependence in the data. The fitted model is (t-statistics are in brackets),

$$x_t = \underset{(15.58)}{0.2x_{t-1}} - \underset{(-0.78)}{0.01x_{t-2}} + \underset{(2.20)}{0.029x_{t-3}} + \underset{(2.23)}{0.029x_{t-4}}$$

$$- \underset{(-0.89)}{0.012x_{t-5}} - \underset{(-1.26)}{0.016x_{t-6}} + e_t$$

Durbin-Watson statistic = 1.9997

Figures 3 and 4 show the plot and histogram of the AR(6) residuals; table 2 presents the mean, standard deviation, skewed and kurtosis statistics.

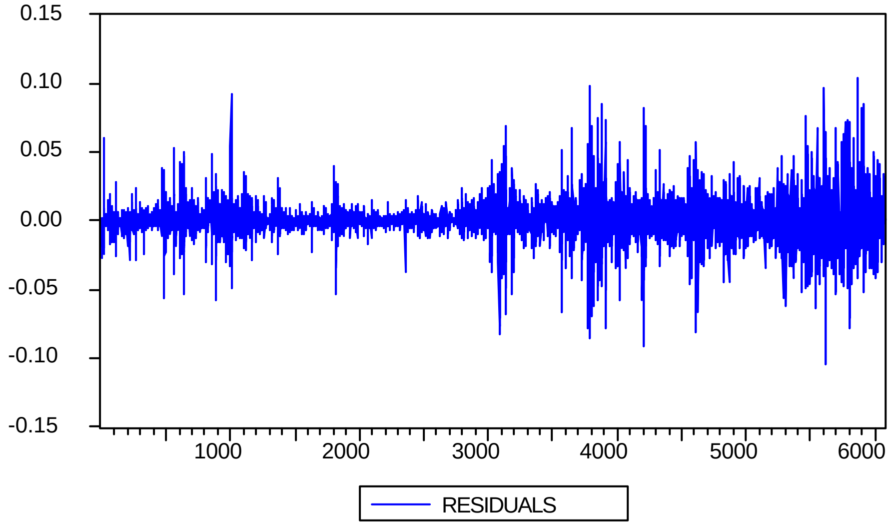

Figure 3: The line graph of the AR(6) residuals of the return series

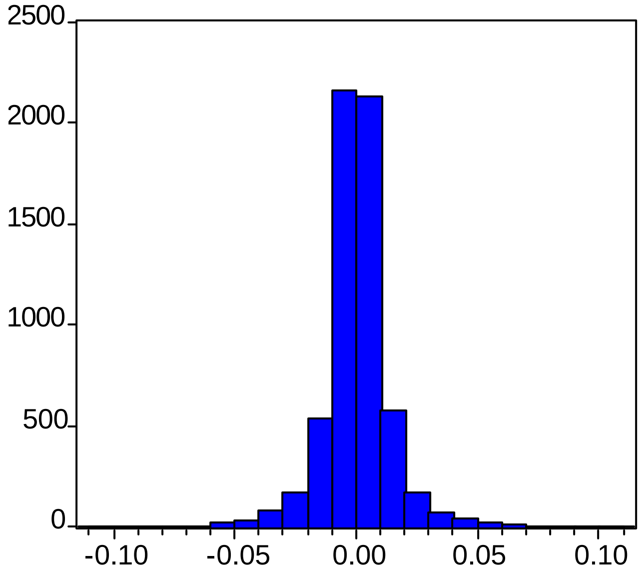

Figure 4: The Histogram normality test of AR(6) residuals of the series

The histogram of the return series shows fat tails and higher peaks than a normal distribution.

No. of obs.	Mean	Stan. Dev.	Skewed	Kurtosis	Z-Value
6070	0.0002	0.0144	0.1975	10.9536	6.283

Table 2: Descriptive statistics for the filtered series

The kurtosis statistic of a normal distribution is 3 and its Z value is ± 2.58 at 1% level of significance. The kurtosis and Z values shown in table 2 indicate that the distribution of the daily logarithmic return on SET index is leptokurtic and non-normal. This finding is consistent with Neuburger and Stokes (1991), Brorsen and Yang (1994), and Hsieh (1991) among others and provides us with the basis for using nonparametric techniques.

Three diagnostic approaches were applied to ensure the filtering of the data or test for the existence of a structure in the residuals. First, the Ljung-Box Q-statistic for testing general serial correlation was computed for lags up to 100. The result shows large and significant Q-statistic from lag 13 up to lag 100, an indication that high order serial autocorrelations are present in the series. Second, a Breusch-Godfrey serial correlation test was run for up to lag 50 and was found significant autocorrelation disturbances for lags 13, 1 4, 29, and 35. Third, the presence of an autoregressive conditional heteroskedasticity (ARCH) process in the series was tested for lags 1, 2, 3, and 4. The result of this test shows significant coefficient and indicates the presence of an ARCH process.

C) <u>Application of Nonlinear Techniques to the Filtered Data</u>

Table 3 gives the estimated correlation dimension (D_m) for the daily logarithmic return series, the filtered series, and the shuffled data.

Embedding Dim.	D_m (original series)	D_m (filtered series)	D_m (Shuffled data)
1	1.017	1.033	1.034
2	2.063	2.043	2.068
3	2.901	2.894	3.001
4	3.565	3.577	3.779
5	4.169	4.192	4.307
6	4.334	4.345	4.836
7	4.901	4.911	5.230
8	5.075	5.068	5.602
9	5.008	5.034	5.781
10	5.407	5.420	5.962

Table 3: Correlation dimension, D_m, (time delay, tau= 1)

First, Brock (1988)'s residual test for chaos was applied by comparing the estimated correlation dimensions of the unfiltered series with the corresponding estimates for the filtered series. There are no clear differences between the two sets of D_m estimates, which indicates the time series passes Brock (1988)'s residual test for the presence of nonlinear chaotic process. Second, the research applied Scheinkman and LeBaron (1989)'s data shuffling procedure that involves generating a shuffled series from the filtered data, calculating the correlation dimensions of the shuffled series and comparing them with those of the filtered data. Again as shown in table 3, there is an apparent increase in the estimated dimensions for the shuffled series. This noticeable increase in the correlation dimensions suggests that the series is chaotic and the shuffling procedure upsets the chaotic structure underlying the time series.

The results of Brock (1988) residual test and Scheinkman and LeBaron (1989) diagnostic procedure support the presence of a low-dimensional nonlinear dynamic or chaotic behavior. This is consistent with the sign of the largest Lyapunov exponent (λ), estimated as $\lambda = 0.529$. A positive Lyapunov exponent implies a sensitive dependence on initial condition; a necessary condition for a chaotic process. Nonetheless, there are indications that point to the absence of a low-dimensional chaotic process or any form of dependence in the observation, be it linear or nonlinear.

First, the most important thing to notice in table 3 is the failure of the estimates of the correlation dimensions for all the series to saturate up to 10 embedding dimensions (with tau = 1). The correlation dimensions of chaotic processes increase with the embedding dimension but up to a certain point after which they remain approximately constant. Second, the BDS statistic for testing the hypothesis the data are IID is less than 1.96, the critical value at the 5% level of significance, for up to 10 embedding dimensions. Therefore, the hypothesis of an IID process cannot be rejected for the time series. Third, the research applied the R/S range analysis and obtained a negative Hurst exponent (H = -0.002). This is rather unusual and perhaps unlikely in a time series. Generally, a value less than H = 0.5 indicates an antipersistent process. Although a negative Hurst exponent is an unusual phenomenon it implies a point attractor, an infinite regress like a black hole[3].

V) **Conclusion**

The purpose of this research was to test the daily return on SET index for nonlinear dynamic behavior, or more specifically, low-dimensional chaotic

[3] The author acknowledges Edgar E. Peters' explanation of the possible implications of a negative Hurst exponent.

behavior. It applied simple descriptive statistical analysis, Brock (1988) residual test, Scheinkman and LeBaron (1989), and rescaled range analysis (R/S) to the data and estimated their correlation dimensions, BDS statistic, and largest Lyapunov exponent.

The results of the study show the distribution of the daily return on SET is non-normal and leptokurtic. This justifies the use of nonparametric tools for the analysis of the data. Brock (1988) and Scheinkman and LeBaron (1989) tests suggest the data are low dimensional chaos; the largest Lyapunov exponent is positive; the correlation dimension fails to saturate up to ten embedding dimensions; the Hurst exponent is small and negative and the BDS statistic is lower than the critical value. The BDS statistic implies, contrary to the findings of residual tests for the presence of serial autocorrelations and ARCH processes, the data are independent and identically distributed. It is possible that the presence of high order autocorrelation revealed by the Ljung-Box Q-statistics and Breusch-Godfrey serial correlation LM test and autoregressive conditional heteroskedasticity (ARCH) process shown by the ARCH test caused a bias in the Hurst exponent.

Notwithstanding the unusual negative exponent, the results of this research suggest that the return series on the SET is high order chaos, which for forecasting purpose is not different from randomness. Indeed, as Hsieh (1991, pp. 9) points out if the system is governed by a “highly complex chaotic process we may never be able to detect it using finite amounts of data.” A high order chaos implies that the series is the result of a complex relationship among many variables, and thus, unpredictable.

Although the estimated parameters are higher, the result of this research is consistent with Khanthavit (1995). The application of advanced noise-reducing techniques and effective ways of eliminating or at least reducing significantly high order serial autocorrelation are recommended for further research to reduce the effects on nonlinear dynamic tests.

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